

# Comparison between Pole Region Design and Model Reference Control for Multivariable TS Fuzzy Systems

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## Abstract

This paper deals with the comparison of two model-based design methods for multivariable systems in the Takagi-Sugeno form. Model-based means that the multivariable control law is synthesized based on a given mathematical process model and a formal description of the desired control properties. Examined are two methods in which the desired characteristics of the control loop are specified by either a parameterized pole region or a closed-loop reference model, which results in different LMI formulations. In particular, the various LMI-based criteria are discussed, and an illustrative example demonstrates their applicability.

## 1 Introduction

In general, the systematic design of a model-based controller requires a suitable process model and formal description of the desired closed-loop dynamics. Especially for tasks like multivariable control of nonlinear systems, this paradigm is widely used in the control community. A major part of the model-based design of fuzzy controllers (Takagi-Sugeno form) examines the stabilization

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of systems particular stability relaxations and considers less the consideration and guarantee of control performance. This is evident also from the focus of the review papers [3] and [6]. In [3], a systematization of methods for proving global-asymptotic stability (GAS) by Lyapunov inspired verification methods based on common quadratic, piecewise quadratic or fuzzy Lyapunov functions is proposed. An investigation of methods for the consideration of the controller performance is not presented. The overview of the controller design in [6] is also limited to the proof of GAS. Nevertheless, there are approaches to LTI systems that explicitly include a formal description of the desired performance of multivariable control problems. However, these have not yet been sufficiently elaborated for the TS framework. One can distinguish two relevant methods: First, by applying the LMI region design method of LTI systems [1],[2] to the class of Takagi-Sugeno (TS) fuzzy systems, it is possible to consider the desired control behavior via the parameters of pole regions in the design. Here, a pole region is a subregion of the complex left-half plane which is described by a LMI region [2]. Thanks to the convex combination of the total number of  $N_r$  submodels, the transfer of LMI regions to TS systems is straightforward. Instead of  $n = 2$ , a maximum of  $n = N_r^2 + 1$  LMIs must be considered. The last mentioned number is calculated very conservatively without utilizing the submodels' double sum symmetry or common matrices. Note, if the symmetry of the double sum is utilized, then just  $n = N_r(N_r + 1)/2 + 1$  are considered in the design. Further details are described in [10].

Second, the desired closed-loop performance of the control can also be specified by a reference model. It is used to describe a desired closed loop input/output characteristic that is directly integrated into the design. The method studied in this paper in the context of TS systems has previously been presented for the class of LPV systems in [4]. Upcoming work will investigate the additional degrees of design freedom provided by TS systems compared to LPV systems. The objective of this paper is first to present the formal design procedures for the fuzzy system framework. The applicability is demonstrated by a mathematical example, which simplifies the dynamics and requirements of power plant units in a coordinated power plant network related to the novel concept of Dynamic Virtual Power Plants (DVPP) proposed in [5].

The paper is structured as follows: Section 2 briefly discusses the LMI formulation for the pole region specification. The controller structure for TS

fuzzy model matching with a reference model and integral state controller as PDC (parallel distributed compensator) structure is presented in Section 3. For illustration, a mathematical example is used in Section 4 that abstractly represents an electrical generation unit in an interconnected power system.

## 2 TS control design by specification of closed-loop pole region

Given is the control law

$$u = \sum_{i=1}^{N_r} h_i(z) (-K_{x,i} x + K_{I,i} x_I), \quad x_I = \int_0^t (y^r(\tau) - y(\tau)) d\tau, \quad (1)$$

with the introduced auxiliary state vector

$$x_I = \int_0^t (y^r(\tau) - y(\tau)) d\tau, \quad (2)$$

where  $y^r$  denotes the external reference value. The control law can be formulated in compact form

$$u = - \sum_{i=1}^{N_r} h_i(z) \underbrace{(K_{x,i}, -K_{I,i})}_{\tilde{K}_i} \tilde{x} \quad (3)$$

by introducing the extended state vector  $\tilde{x}^T = (x^T, x_I^T)$ . In the TS framework the controlled plant is represented by the standard form

$$\dot{x} = \sum_{i=1}^{N_r} h_i(z) (A_i x + B_i u), \quad y = \sum_{i=1}^{N_r} h_i(z) C_i x \quad (4)$$

with the convex sum condition  $0 \leq h_i \leq 1$  and  $\sum_{i=1}^{N_r} h_i(z) = 1 \forall z$ . The augmented design model results from (2) and (4)

$$\dot{\tilde{x}} = \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} A_i & 0 \\ -C_i & 0 \end{pmatrix}}_{\tilde{A}_i} \tilde{x} + \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} B_i \\ 0 \end{pmatrix}}_{\tilde{B}_i} u + \underbrace{\begin{pmatrix} 0 \\ I \end{pmatrix}}_{\tilde{E}} y^r. \quad (5)$$

By substituting  $u$  in (5) by the control law (3) follows the augmented closed-loop dynamics

$$\dot{\tilde{x}} = \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} h_i(z) h_j(z) (\tilde{A}_i - \tilde{B}_i \tilde{K}_j) \tilde{x} + \tilde{E} w. \quad (6)$$

Note the different indexing of the plant model and the controller due to different input matrices resulting in the double sum. The design for determining the controller gain  $\tilde{K}_j$  is based on the specification of a desired pole region  $S(\alpha, r, \theta)$  of all eigenvalues  $\lambda_{ij}$  given by

$$\lambda_{ij} = \text{eig}(\tilde{A}_i - \tilde{B}_i \tilde{K}_j) \quad \text{for } i, j = 1, \dots, N_r. \quad (7)$$

Thereby the pole region is defined by the parameters  $\alpha, r$ , and  $\theta$  as shown in Figure 1. The final design of the controller is based on the following LMI formulation: All eigenvalues  $\lambda_{ij}$  (7) of the closed-loop system (6) with the PDC control law (3) are located in the given pole region  $S(\alpha, r, \theta)$  if there exists a common symmetric matrix  $X \succ 0$  and  $M_j$ ,  $j = 1, 2, \dots, N_r$  as feasible solution of the LMI problem

$$\begin{aligned} \Gamma_{ij}^1 &= \tilde{A}_i X + X \tilde{A}_i^T - \tilde{B}_i M_j - M_j^T \tilde{B}_i^T + 2\alpha X, \\ \Gamma_{ij}^2 &= \begin{pmatrix} (\tilde{A}_i X + X \tilde{A}_i^T - \tilde{B}_i M_j - M_j^T \tilde{B}_i^T) \sin \theta & (\tilde{A}_i X - X \tilde{A}_i^T - \tilde{B}_i M_j + M_j^T \tilde{B}_i^T) \cos \theta \\ (X \tilde{A}_i^T - \tilde{A}_i X - M_j^T \tilde{B}_i^T + \tilde{B}_i M_j) \cos \theta & (\tilde{A}_i X + X \tilde{A}_i^T - \tilde{B}_i M_j - M_j^T \tilde{B}_i^T) \sin \theta \end{pmatrix}, \\ \Gamma_{ij}^3 &= \begin{pmatrix} -rX & \tilde{A}_i X - \tilde{B}_i M_j \\ X \tilde{A}_i^T - M_j^T \tilde{B}_i^T & -rX \end{pmatrix} \end{aligned} \quad (8)$$

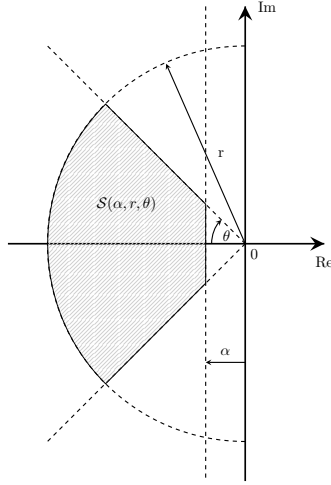


Figure 1: Parameterized pole region  $S(\alpha, r, \theta)$

with the relaxed conditions

$$\begin{aligned} \Gamma_{ij}^k(X, M_j) + \Gamma_{ji}^k(X, M_i) &< 0, \\ \Gamma_{ii}^k(X, M_i) &< 0, \quad k = 1, 2, 3 \quad \text{for all } i = 1, 2, \dots, N_r, \quad j = i + 1, i + 2, \dots, N_r \\ \text{s.t. } h_i(z)h_j(z) &\neq 0, \exists z \end{aligned}$$

So that, the feedback gains of the control law (3) can be obtained

$$\tilde{K}_j = M_j X^{-1}, \quad j = 1, 2, \dots, N_r. \quad (9)$$

An application of this concept in the stability and control of renewable-energy power plants has been demonstrated, i.e., for wind turbines [8] and PV power plants [9].

### 3 TS controller design by model matching using reference model specification

In comparison to the specification of a pole region as shown in Figure 1, for the second method the closed dynamics is specified by a reference model

$$\dot{x}^r = A^r x^r + E^r w, \quad y = C^r x^r + F^r w. \quad (10)$$

The used process model distinguishes between the controller input  $u$  and an external reference  $w$ :

$$\dot{x} = \sum_{i=1}^{N_r} h_i(z) (A_i x + B_i u + E_i w), \quad y = \sum_{i=1}^{N_r} h_i(z) (C_i x + F_i w). \quad (11)$$

The related the control law is specified as

$$u = - \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} K_{x,i} & K_{x^r,i} & -K_{l,i} \end{pmatrix}}_{\bar{K}_i} \underbrace{\begin{pmatrix} x \\ x^r \\ x_l \end{pmatrix}}_{\bar{x}} \quad (12)$$

with the same auxiliary state vector  $x_l$  proposed in (2). An augmented design model is specified using the extended state vector  $x$  (12) and matching error  $\varepsilon = \dot{x}_l = y^r - y$ :

$$\begin{aligned} \dot{\bar{x}} &= \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} A_i & 0 & 0 \\ 0 & A^r & 0 \\ -C & C^r & 0 \end{pmatrix}}_{\bar{A}_i} \bar{x} + \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} B_i \\ 0 \\ 0 \end{pmatrix}}_{\bar{B}_i} u + \sum_{i=1}^{N_r} h_i(z) \underbrace{\begin{pmatrix} E_i \\ E_i^r \\ F^r - F_i \end{pmatrix}}_{\bar{E}_i} w, \\ \varepsilon &= \underbrace{\begin{pmatrix} -C_i & C^r & 0 \end{pmatrix}}_{\bar{C}_i} \bar{x} + \underbrace{\begin{pmatrix} F^r - F_i \end{pmatrix}}_{\bar{F}_i} w \end{aligned} \quad (13)$$

which results directly from the combination of (2), (11), and (12). By specifying the matching error  $\varepsilon$ , an  $H_\infty$  problem can be formulated to calculate the controller gain  $\bar{K}$ . In this regard, the design objective is to minimize the input-to-output gain denoted as  $\gamma$  with respect to the input  $w$  and output  $\varepsilon$ :

$$\begin{aligned} &\text{minimize} \quad \gamma \\ &\text{subject to} \quad \sup_{\|w\|_2 \neq 0} \frac{\|\varepsilon\|_2}{\|w\|_2} \leq \gamma, \end{aligned} \quad (14)$$

where  $\|\cdot\|_2$  denotes the  $L_2$ -norm. Controller coefficients  $\bar{K}_i, i = 1, \dots, N_r$  from (12) are calculated by solving the associated LMI problem

$$\Gamma_{ij} = \begin{pmatrix} \bar{A}_i X + X \bar{A}_i^T - \bar{B}_i \bar{M}_j - \bar{M}_j^T \bar{B}_i^T & \bar{E}_i & X \bar{C}_i^T \\ \bar{E}_i^T & -\gamma^2 I & \bar{F}_i^T \\ \bar{C}_i X & \bar{F}_i & -I \end{pmatrix}, \quad X \succ 0 \quad (15)$$

where  $X = X^T$  with the relaxed condition

$$\begin{aligned} \Gamma_{ij}(X, M_j) + \Gamma_{ji}(X, M_i) &\prec 0, \\ \Gamma_{ii}(X, M_i) &\prec 0, \text{ for all } i = 1, 2, \dots, N_r, \quad j = i+1, i+2, \dots, N_r. \\ \text{s.t. } h_i(z)h_j(z) &\neq 0, \exists z \end{aligned}$$

## 4 Illustrating example

To illustrate the model matching method, we now examine the following multivariable nonlinear system to be controlled

$$\dot{x} = \begin{pmatrix} -\frac{1}{2} \cos(\frac{\pi}{20} x_1) & \frac{1}{3} \\ \frac{1}{3} & -\frac{4}{5} \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} w, \quad y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x. \quad (16)$$

with  $x = (x_1, x_2)^T$ ,  $w = (\Delta f, \Delta v)^T$ , and  $y = (\Delta p, \Delta q)^T$ . The plant model is intended to represent a generating unit in a power system with the measured grid frequency  $\Delta f$  and voltage change  $\Delta v$  at the point of common coupling (PCC). The plant output consists of the change of active power  $\Delta p$  and reactive power  $\Delta q$  injected into the grid. A controller (12) is to be found, which should match the plant model (16) to the reference model

$$\dot{x}^r = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{4}{5} \end{pmatrix} x^r + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} w, \quad y^r = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x^r. \quad (17)$$

The reference model (17) is thus designed to reproduce the desired dynamic characteristics of a generating unit for grid control purposes. To be able to use the LMI formulation (15) for the controller design, the plant model must first be converted into a Takagi-Sugeno form. For this we apply the sector nonlinearity approach [7], where a bounded function  $f(x) \in [\min(f), \max(f)]$  is described by

$$f(x) = \underbrace{\frac{\max(f) - f(x)}{\max(f) - \min(f)}}_{h_1(x)} \min(f) + \underbrace{\frac{f(x) - \min(f)}{\max(f) - \min(f)}}_{h_2(x)} \max(f) \quad (18)$$

Considering the practical limitation of the state space  $x_1 \in [-4, 4]$ , the min-max values for the example (16) with  $f(x) = \cos(\frac{\pi}{20} x_1)$  are calculated by

$$\min(f) = \cos(\frac{\pi}{20} \max(x_1)), \quad \max(f) = \cos(\frac{\pi}{20} 0) = 1. \quad (19)$$

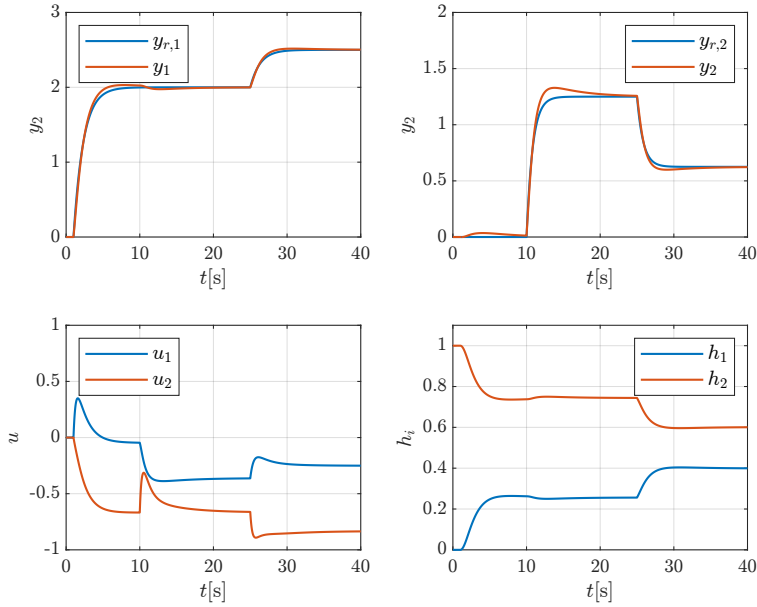


Figure 2: Simulation results using the model matching method

This results in the matrices of the submodels

$$A_1 = \begin{pmatrix} -\frac{1}{2} \max(f) & \frac{1}{4} \\ \frac{1}{3} & -\frac{4}{5} \end{pmatrix}, \quad A_2 = \begin{pmatrix} -\frac{1}{2} \min(f) & \frac{1}{4} \\ \frac{1}{3} & -\frac{4}{5} \end{pmatrix} \quad (20)$$

and by adding the membership functions function (18) we obtain

$$\dot{x} = \sum_{i=1}^2 h_i(z) A_i x + B u, \quad y = C x \quad (21)$$

with the common  $B = B_1 = B_2$  and  $C = C_1 = C_2$ . After this preliminary work is done, the augmented model according to (13) can now be created, and the LMI problem (15) formulated. The simulation result for a unit step of  $\Delta f$  at  $t = 1$ s and of  $\Delta v$  at  $t = 10$ s is given in Figure 2.



## 5 Conclusion

Two methods for specifying the desired closed-loop dynamic for multivariable Takagi-Sugeno systems were presented. The necessary design steps were introduced and compared. A simplified example from the area of power plant control was used for the model matching procedure. In ongoing work, we are investigating how to integrate time-variable reference models into the design. The novel concept would address new design possibilities for fast coordination of power plants in interconnection with dynamic participation factors proposed in [4].

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## References

- [1] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, Philadelphia, PA, USA: SIAM, 1994.
- [2] M. Chilali and P. Gahinet.  *$H_\infty$  design with pole placement constraints: An LMI approach*. IEEE Transactions on Automatic Control, vol. 41, no. 3, pp. 358-367, 1996.
- [3] G. Feng, *A Survey on Analysis and Design of Model-Based Fuzzy Control Systems*, IEEE Transaction on Fuzzy Systems, vol. 14, no. 5, October 2006.
- [4] V. Häberle, M. W. Fisher, E. Prieto-Araujo, F. Dörfler. *Control Design of Dynamic Virtual Power Plants: An Adaptive Divide-and-Conquer Approach*, IEEE Transaction on Power Systems, vol. 37, no. 5, September 2022.
- [5] B. Marinescu, O. Gomis-Bellmunt, F. Dörfler, H. Schulte and L. Sigríst. *Dynamic Virtual Power Plant: A New Concept for Grid Integration of Renewable Energy Sources*, IEEE Access, September 2022, doi: 10.1109/ACCESS.2022.3205731.

- [6] A.T. Nguyen, T. Taniguchi, L. Eciolaza, M. Sugeno. *Fuzzy Control Systems: Past, Present and Future*, IEEE Computational Intelligence Magazine, vol: 14, no. 1, February 2019.
- [7] H. Ohtake, K. Tanaka, and H. O. Wang. *Fuzzy Modeling via Sector Nonlinearity Concept*. In Joint 9th IFSA World Congress and 20th NAFIPS International Conference, pp. 127-132, Vancouver, Canada, 2001.
- [8] F. Pöschke, V. Petrovic, F. Berger, L. Neuhaus, M. Hölling, M. Kühn, and H. Schulte, *Model-based wind turbine control design with power tracking capability: a wind-tunnel validation*, Control Engineering Practice, vol. 120, March 2022
- [9] S. Kusche, H. Schulte, *Demanded Power Point Tracking of PV Power Plants without Battery Energy Storage*, Proc. 31st Workshop on Computational Intelligence, pp. 169-187, 2022.
- [10] K. Tanaka and H.O. Wang. *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, Inc, 2001.